## Theory of machinery

## Chapter three

## Velocity analysis

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## Velocity analysis

As in position analysis, it is important in many cases to analyze the velocity of moving links.

Before start with velocity analysis, let us review some of the basic concepts we will need it in future. Let us assume a body rotating about certain axis. The angular velocity ( $\omega$ ) for this body is found by deriving the angular dimension $\Theta$ with respect to time, mathematically:-

$$
\omega=\frac{d \theta}{d t}=\dot{\theta}
$$

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## Velocity analysis

Due to the fixed links lengths, the velocity components is reduced from normal (radial) and tangential components to tangential component ( $\mathbf{V}_{\mathrm{T}}$ )only. As you remember, velocity is a vector quantity and $V_{T}$ is perpendicular to $\mathbf{R}$ as shown The relation between $\mathbf{V}$ and $\boldsymbol{\omega}$ is : $\mathbf{V}=\mathrm{R} . \boldsymbol{\omega}$.

This relation is for the magnitude.
The polygon method depends on this relation to find the angular velocity after finding the tangential one.


## Velocity analysis

Velocity polygon method

## 4-bar mechanism

For a known four-bar mechanism, in a given configuration and for a known angular velocity of the crank, $\omega_{2}$, we want to determine $\omega_{3}$ and $\omega_{4}$. In this example we assume $\omega_{2}$ is CCW. For the position vector loop equation


## Velocity analysis

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For the position vector loop equation:

$$
R_{2}+R_{3}-R_{4}-R_{1}=0---(1)
$$

the velocity equation is

$$
V_{2}+V_{3}-V_{4}=0--(2)
$$



As mentioned before, the velocity component is tangential and Eq. 2 becomes:

$$
\omega_{2} R_{2}+\omega_{3} R_{3}-\omega_{4} R_{4}=0
$$

Eq.1. is sometimes called vector enclosure equation because the velocity vector summation ends with zero.

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Velocity polygon method:-
As illustrated in Eq.2, the velocity enclosure equation
Can be represented in the following vector diagram


$$
V_{2}+V_{3}-V_{4}=0--(2)
$$

So, our task is to draw this diagram or polygon.
Remember, the velocities vectors are perpendicular to the position vectors.

## Velocity analysis

## Velocity polygon method

## 4-bar mechanism

$\square$ To start, select a reference zero velocity point ( $\mathrm{O}_{\mathrm{V}}$ )
$\square$ Draw a perpendicular line to vector $R_{2}$ from $O_{V}$ as shown in the next figure. Note that vector $\mathrm{V}_{2}$ can be

drawn directly because its angle is determined by the line drawn previously and its magnitude is $\boldsymbol{\omega}_{\mathbf{2}} \mathbf{R}_{\mathbf{2}}$

DNote that the direction of $\mathbf{V}_{\mathbf{2}}$ is determined b! rotating $R_{2}$ in the same direction of $\omega_{2}$.


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From $A$ draw a line perpendicular to $\boldsymbol{R}_{\mathbf{3}}, \mathbf{V}_{\mathbf{3}}$ must reside on this line.

From $O_{V}$ draw a line perpendicular to $\boldsymbol{R}_{\mathbf{4}} . \mathbf{V}_{\mathbf{4}}$ must reside on this line.

Construct vectors $\mathbf{V}_{\mathbf{3}}$ and $\mathbf{V}_{\mathbf{4}}$.
Determine the magnitude of V3 from the polygon.
Compute $\omega_{3}=V_{3} / R_{3}$. Determine the direction of $\omega_{3}$. In this example it is CW since $\mathbf{R}_{\mathbf{3}}$ must rotate $90^{\circ} \mathbf{C W}$ to line up with $\mathbf{V}_{\mathbf{3}}$


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Determine the magnitude of VB from the polygon. Compute $\omega_{4}=V_{4} / R_{4}$. Determine the direction of $\omega_{4}$. In this example it is CCW since $R_{4}$ must rotate $90^{\circ} \mathrm{CCW}$ to line up with $\mathrm{V}_{4}$.


## Velocity analysis

## Slider crank mechanism

This slider-crank mechanism in the given configuration has a known angular velocity of the crank, $\omega_{2}$. We want to determine $\omega_{3}$ and the velocity of the slider block. In this example we assume $\omega_{2}$ is CCW.


## Velocity analysis

## Slider crank mechanism

The position vector loop equation is:

$$
R_{A O 2}+R_{B A}-R_{B O 2}=0
$$

The velocity (loop) equation is expressed as

$$
\begin{gathered}
V_{A}+V_{B A}-V_{B}=0 \\
\omega_{2} R_{A O 2}+\omega_{3} R_{B A}-\omega_{4} V_{B}=0
\end{gathered}
$$




## Velocity analysis

## Slider crank mechanism



